C 21297 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MAT 4C 04—MATHEMATICS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write Laplace transform of $f''(t)$.
2. Write the general form of second order linear ODE.
3. Find L (e^{at}).
4. What is unit step function ? Give an example.
5. Give a formula for an error for Simpson's rule.
6. Find the fundamental period for sin x.
7. Find Wronskian of $\cos \omega x$ and $\sin \omega x$.
8. What is particular solution of an ODE ?
9. What do you mean by an even function give example.
10. Write the 1-dimensional Heat equation.
11. State second shifting theorem for Laplace transfom.
12. Solve $y'' - y = 0$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find a basis for the solution of the differential equation $y'' - y = 0$.
14. Show that Laplace transform is a linear operator.

Turn over

15. Solve the initial value problem $y'' + 2y' + 2y = 0,\ y(0) = 1,\ y'(0) = -1$.

16. Factor $(D^2 + 6D + 13I)y = 0$ and solve it.

17. Find $L^{-1}\left(\dfrac{\sqrt{8}}{\left(s+\sqrt{2}\right)^3}\right)$.

18. If $f(x)$ is a periodic function of x of period p, show that $f(ax),\ a \neq 0,$ is a periodic function of x of period $\dfrac{p}{a}$.

19. Find the Fourier cosine transform of $e^{-ax}, a > 0$.

20. Find an ODE for the basis e^{2x}, e^{x}.

21. Solve $y'' - y = t,\ y(0) = 1,\ y'(0) = 1$ by applying Laplace transform.

22. Check whether the functions 5 sin x cos x, 3 sin 2x, $x > 0$ are linearly independent.

23. Find solutions u of the PDE $u_{xx} - u = 0$.

24. Find an upper bound for the error incurred in estimating $\int_0^2 5x^4\,dx$ using Simpson's rule with $n = 4$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find $L^{-1}\left(\dfrac{1}{\left(s^2+w^2\right)^2}\right)$.

26. Find solution of the initial value problem $y'' + 4y = 16\cos 2x,\ y(0) = 0,\ y'(0) = 0$.

27. Find the Laplace transform of $e^{-\alpha t}\cos\beta t$.

28. Find the inverse transform $f(t)$ of $F(s) = \frac{e^{-s}}{s^2+\pi^2} + \frac{e^{-2s}}{s^2+\pi^2} + \frac{e^{-3s}}{(s+2)^2}$.

29. Find a general solution of the differential equation :

$y'' + 3y' + 2y = 30e^{2x}$.

30. Find the Fourier series of $f(x) = \begin{cases} -k, \text{ if } -\pi < x < 0 \\ k, \text{ if } 0 < x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$.

31. How many subdivisions should be used in the Trapezoidal Rule to approximate $\ln 2 = \int_1^2 \frac{1}{x}\,dx$ with an error whose absolute value is less than 10^{-4}.

32. Given $y' = x(1-y), y(1) = 0, dx = 0.2$. Find the first three approximations by improved Euler method. Compare with exact solution.

33. Evaluate $\int_{-1}^{1}(1+x^2)\,dx$ with $n = 4$ steps and find an upper bound for $|E_s|$ using Simpson's rule.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $y'' + 3y' + 2y = 1$ if $0 < t < a$ and 0 if $t > a$; $y(0) = 0, y'(0) = 0$.

35. Find the Fourier series of $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x+2\pi) = f(x)$. Hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots = \frac{\pi^2}{12}$.

36. Solve the integral equation $y(t) - \int_0^t (1+\tau)\,y(t-\tau)\,d\tau = 1 - \sinh t$.

(2 × 10 = 20 marks)