

C 20647

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Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Questions)*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

- Find the domain and range of the function $f(x, y) = x + 3y - 1$.
- Evaluate $\lim_{(x,y,z) \rightarrow (\frac{\pi}{2}, 0, 1)} \frac{e^{2y} (\sin x + \cos y)}{1 + y^2 + z^2}$.
- Find f_x and f_y if $f(x, y) = x \cos xy^2$.
- Find the directional derivative of $f(x, y) = x^2 \sin 2y$ at $(1, \frac{\pi}{2})$ in the direction of $\vec{v} = 3\hat{i} - 4\hat{j}$.
- Find $\nabla f(x, y, z)$ if $f(x, y, z) = x^2 + y^2 - 4z$ and find the direction of maximum increase of f at the point $(2, -1, 1)$.
- Find the relative extrema of the function $f(x, y) = x^2 + y^2 - 2x + 4y$.
- Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ above the rectangular region given by $0 \leq x \leq 1, 0 \leq y \leq 2$.
- Find the mass of the triangular lamina with vertices $(0, 0), (0, 3)$ and $(2, 3)$ given that the density at (x, y) is $\rho(x, y) = 2x + y$.

Turn over

9. Find the Jacobian for the change of variables defined by $x = r \cos \theta, y = r \sin \theta$.
10. Evaluate $\iint_R 2x - y \, dA$ where R is the region bounded by the parabola $x = y^2$ and the line $x - y = 2$.
11. Find whether the vector field $\vec{F} = x^2y \hat{i} + xy \hat{j}$ is conservative.
12. State Green's theorem.
13. Find a parametric representation for the cone $x = \sqrt{y^2 + z^2}$.
14. Find the surface area of the torus given by $\vec{r}(u, v) = (2 + \cos u) \cos v \hat{i} + (2 + \cos u) \sin v \hat{j} + \sin u \hat{k}$ where the Domain D is given by $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.
15. Compute $\iint_S \vec{F} \cdot \hat{n} \, dS$ given $\vec{F}(x, y, z) = (x + \sin z) \hat{i} + (2y + \cos x) \hat{j} + (3z + \tan y) \hat{k}$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(10 × 3 = 30 marks)

Section B (Paragraph Questions)

*Answer at least five questions.
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.*

16. Find f_{xyx} and f_{yx} if $f(x, y) = x \cos y + y \sin x$.
17. Find the differential of $w = x^2 + xy + z^2$. Compute the value of dw if (x, y, z) changes from $(1, 2, 1)$ to $(0.98, 2.03, 1.01)$ and compare the value with that of Δw .
18. Find the equation of the tangent plane and normal line to the surface $x^2 - 2y^2 - 4z^2 = 4$ at $(4, -2, -1)$.
19. Find the relative extrema of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$.
20. Find the volume of the solid region bounded by the paraboloid $z = 4 - x^2 - 2y^2$ and the xy -plane.

21. Evaluate $\int_0^2 \int_0^x \int_0^{x+y} e^x (y+2z) dz dy dx$.
22. Determine whether the vector field $\vec{F} = e^x (\cos y \hat{i} - \sin y \hat{j})$ is conservative. If so, find a potential function for the vector field.
23. Evaluate $\oint_C (e^x + y^2) dx + (x^2 + 3xy) dy$, where C is the positively oriented closed curve lying on the boundary of the semi annular region R bounded by the upper semicircles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ and the x -axis.

(5 × 6 = 30 marks)

Section C (Essay Questions)

*Answer any two questions.
Each question carries 10 marks.*

24. (a) Sketch the graph of $f(x, y) = 9 - x^2 - y^2$.
- (b) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist.
25. (a) Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.
- (b) Find the minimum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.
26. Let R be the region bounded by the square with vertices (0,1), (1,2), (2,1) and (1,0). Evaluate $\iint_R (x+y)^2 \sin^2(x-y) dA$.
27. Let $\vec{F} = (x, y, z) = 2xyz^2 \hat{i} + x^2z^2 \hat{j} + 2x^2yz \hat{k}$.
- (a) Show that \vec{F} is conservative and find a scalar function f such that $\vec{F} = \nabla f$.
- (b) If \vec{F} is a force field, find the work done by \vec{F} in moving a particle along any path from (0, 1, 0) to (1, 2, -1).

(2 × 10 = 20 marks)