C 4388-B **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Prove that $\cosh^2 x - \sinh^2 x = 1$.

2. Find the Cartesian form of the polar equation $r = \dfrac{8}{1 - 2\cos\theta}$.

3. Find the slope of the line tangent to the graph of $r = 3\cos^2 2\theta$ at $\theta = \pi/6$.

4. Evaluate $\int \sinh^2 x dx$.

5. Show that $\lim\limits_{n\to\infty} \dfrac{2n}{n^2+1} = 0$.

6. Test the convergence of the series $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{4}} - \dfrac{1}{\sqrt{8}} + \dfrac{1}{\sqrt{16}} \ldots$

7. Compute $\|\cos x\|$ in $C[0, 2\pi]$.

8. Examine whether the set of vectors $u_1 = \langle 1,2,3 \rangle, u_2 = \langle 2,4,3 \rangle$, and $u_3 = \langle 3,2,1 \rangle$ is linearly independent or not.

9. Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$.

10. Find the determinant of the matrix $C = \begin{bmatrix} -1 & 2 & 9 \\ 2 & -4 & -18 \\ 5 & 7 & 27 \end{bmatrix}$.

Turn over

11. Show that $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is an orthogonal matrix.

12. Find the eigen values of the matrix $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$.

14. Find the equation of the tangent line when $t = 1$ for the curve $x = t^4 + 2\sqrt{t}, y = \sin(t\pi)$.

15. Find the length of the perimeter of the cardioid $r = a(1 - \cos\theta)$.

16. Use the Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. Compare the estimate with the exact value of the integral.

17. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

18. Show that the set B = {(1, 2, 1), (2, 1, 0), (1,–1, 2)} is a basis for R^3.

19. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Evaluate $\int_{1}^{\infty} \frac{ln x}{x^2}\, dx,$ if it exists.

(b) Find the area of the region shared by the cardioids $r = 2(1 + \cos\theta)$ and $r = 2(1 - \cos\theta)$.

21. (a) Solve :

$$x_1 + x_2 + x_3 + x_4 = 0$$
$$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$$
$$2x_1 + x_3 - x_4 = 0.$$

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

(1 × 11 = 11 marks)