201801

C 22093

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Name..... Reg. No.....

# SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2C 01—MATHEMATICS—2

(2021 Admissions)

Time : Two Hours

Maximum : 60 Marks

#### Section A

Answer at least **eight** questions. Each question carries 3 marks. All questions can be attended. Overall Ceiling 24.

- 1. Find the inverse of  $f(x) = \frac{2x-3}{5x-7}$ , where the domain of f excludes  $x = \frac{7}{5}$ .
- 2. Find the Cartesian form of the polar equation  $r = \sin 2\theta$ .
- 3. Express the number  $\operatorname{coth}^{-1}(5/4)$  in terms of natural logarithms.
- 4. Prove that  $tanh^2x + sech^2 x = 1$ .
- 5. Show that the series  $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$  converges and also find its sum.
- 6. Find the norm of the vector (3, 4, 0, 1, -1). Also normalize the vector.

7. Determine the radius of convergence of  $\sum_{k=0}^{\infty} \frac{k^5}{(k+1)!} x^k$ .

- 8. Find a basis and then give the dimension of solution space of.
- 9. Find the inner product of the vectors  $\mathbf{a} = \langle 1, 2, 3 \rangle$  and  $\mathbf{b} = \langle 0, -2, 1 \rangle$  in  $\mathbb{R}^3$ . Are the vectors orthogonal ?

10. Show that  $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$  is an orthogonal matrix.

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11. If  $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$  find  $A^3$  using Cayley Hamilton theorem.

12. Find the inverse of the 2 × 2 matrix  $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ .

 $(8 \times 3 = 24 \text{ marks})$ 

#### **Section B**

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Answer at least **five** questions. Each question carries 5 marks. All questions can be attended. Overall Ceiling 25.

- 13. Find the length of the graph of  $f(x) = (x-1)^{3/2} + 2$  on [1, 2].
- 14. Diagonalize the matrix  $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$ .
- 15. Find the length of the perimeter of the cardioid  $r = \alpha(1 + \cos \theta)$ .
- 16. Find an approximation value of  $\int_{0}^{1} x^{2} dx$  by Simpson's rule with n = 10.
- 17. Expand log x in ascending powers of x 1 as for the term containing  $(x 1)^4$ .
- 18.  $B_1 = \{u_1, u_2, u_3\}$ , where  $u_1 = \langle 2, -1, 1 \rangle$ ,  $u_2 = \langle 1, 5, 1 \rangle$ ,  $u_3 = \langle 0, 1, 2 \rangle$ , is a basis for  $\mathbb{R}^2$ . Transform it into an orthonormal basis  $B_2 = \{w_1, w_2, w_3\}$ .

19. Find the rank of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$  by reducing it to the echelon form.

 $(5 \times 5 = 25 \text{ marks})$ 

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## Section C

## Answer any **one** question. The question carries 11 marks.

- 20. (a) Find the area of the region shared by the circles r = 1 and  $r = 2 \sin \theta$ .
  - (b) Show that the set  $B = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$  is a basis for  $\mathbb{R}^3$ .
- 21. (a) Using Gauss-Jordan elimination method, solve the system of equations :

(b) Find the eigen values of A =  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ .

(1 × 11 = 11 marks)