FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time: Three Hours

Maximum: 30 Weightage

Section A

8 short questions answerable within 7.5 minutes. Answer **all** questions, each question carries 1 weightage.

- 1. Write down the relation between cartesian coordinate system and spherical polar co-ordinate system.
- 2. What are Tensors? Define the rank of the tensor.
- 3. Write the expression for Fourier co-efficients.
- 4. What do you mean by a self-adjoint differential equation?
- 5. Write down the Rodrigues formula of Laguerre function and obtain $L_1(x)$ from Rodrigues formula.
- 6. Define a unitary matrix with an example.
- 7. Obtain the recurrence formula, $H'_{n}(x) = 2nH_{n-1}(x)$ from generating function.
- 8. Explain the convolution property of Fourier transform with an example.

 $(8 \times 1 = 8 \text{ weightage})$

Section B

4 essay questions answerable within 30 minutes. Answer any **two** questions, each question carries weightage 5.

- 9. Discuss the orthogonality property of Legendre polynomials.
- 10. Explainthe Frobenius' method of finding solution to homogenous differential equation of second order.

Turn over

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- 11. Prove that $\nabla \cdot r^n \hat{r} = (n+2)r^{n-1}$.
- 12. State and prove the Quotient rule in tensors.

 $(2 \times 5 = 10 \text{ weightage})$

Section C

2

7 problems answerable within 15 minutes. Answer any **four** questions, each question carries weightage 3.

- 13. Express the spherical polar unit vectors in terms of cartesian unit vectors.
- 14. Show that $\Gamma(p+1) = p \Gamma(p)$.
- 15. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.
- 16. Find Laplace transform of the function, $f(t) = t^n$.
- 17. Define spherical Bessel function. Obtain the expression for $j_1(x)$.
- 18. Find the Fourier transform of the normalized Gaussian distribution

$$f(t) = \frac{1}{\tau \sqrt{2\pi}} \exp\left(\frac{-t^2}{2\tau^2}\right), -\infty < t < \infty, \text{ where } \tau = \Delta t \text{ (root mean square deviation)}.$$

19. A and B are two non-commuting Hermitian matrices: AB - BA = iC. Prove that C is Hermitian.

 $(4 \times 3 = 12 \text{ weightage})$