**D 124472** **(Pages : 3)** **Name..............................**

**Reg. No..............................**

# SECOND SEMESTER (CBCSS-UG) DEGREE SUPPLEMENTARY EXAMINATION, APRIL 2025

Mathematics

MTS 2C 02—MATHEMATICS—2

(2020 Syllabus)

Time : Two Hours Maximum Marks : 60

## Section A

*Answer any number of questions.*
*Each question carries 2 marks. (Ceiling is 20).*

1. Find the polar co-ordinates of $(x, y) = (5, -2)$.
2. State inverse function rule and verify this rule for $y = x^3$.
3. Show that $\tanh^2 x + \operatorname{sech}^2 x = 1$.
4. Show that $\int_0^\infty \frac{\sin x}{(1+x)^2}\,dx$ converges.
5. Sum the series $2 + 4 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$.
6. Test convergence for the series $\sum_{n=1}^{\infty} \frac{4^n + 5^n}{2^n 3^n}$.
7. Show that the vector $p_1(x) = x + 1, p_2(x) = x - 1$ are linearly independent vectors in $P_1$.
8. Find the rank of $A = \begin{bmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ -1 & -1/2 & -3/2 \end{bmatrix}$.
9. Find the values of $\lambda$ which satisfies $\begin{vmatrix} -3-\lambda & 10 \\ 2 & 5-\lambda \end{vmatrix} = 0$.
10. Find the inverse of $A = \begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$.

**Turn over**

11. Let A be a non-singular matrix. If $\lambda$ is an eigenvalue of A with corresponding eigenvector $k$, prove that $1/\lambda$ is an eigenvalue of $A^{-1}$ with the same corresponding eigenvector $k$.

12. Show that the rotation matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

## Section B

*Answer any number of questions.*
*Each question carries* 5 *marks.* (*Ceiling is* 30).

13. Plot the graph of $r = \cos 2\theta$ in the $xy$ plane and discuss its symmetry.

14. Find the length of the parabola $y = x^2$ from $x = 0$ to $x = 1$.

15. For the following table of data evaluate $\int_0^1 f(x)dx$ is Simpson's rule :

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ : | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $f(x)$ : | 0.846 | 0.928 | 0.882 | 0.953 | 1.121 | 1.221 | 1.661 | 2.101 | 2.321 | 3.101 | 3.010 |

16. Show that the geometric series $\sum_{i=1}^{\infty} ar^i$ converges if $|r| < 1$ and diverges if $|r| \geq 1$ and $a \neq 0$.

17. Use the Gram-Schmidt orthogonalization process to transform the vectors $\mathbf{u}_1 = \langle 1,1,1 \rangle$, $\mathbf{u}_2 = \langle 1,2,2 \rangle$ *and* $\mathbf{u}_3 = \langle 1,1,0 \rangle$ and to orthonormal set

18. Balance the chemical equation $C_2H_6 + O_2 \rightarrow CO_2 + H_2O$.

19. Identify the conic section whose equation is $2x^2 + 4xy - y^2 = 1$.

## Section C

*Answer any* **one** *question.*
*The question carries* 10 *marks.*

20. Use Gauss-Jordan elimination to solve

$$\begin{aligned} x_1 + x_3 - x_4 &= 1 \\ 2x_2 + x_3 + x_4 &= 3 \\ x_1 - x_2 + x_4 &= -1 \\ x_1 + x_2 + x_3 + x_4 &= 2 \end{aligned}$$

21. Determine whether the matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$ is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^T AP$

(1 × 10 = 10 marks)