D 120574 **(Pages : 4)** **Name.................................**

Reg. No..............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2025

Mathematics

MTS4B04—LINEAR ALGEBRA

(2019—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer type Question)

Each question carries 2 marks.
All questions can be attended.
Overall ceiling 25.

1. Give an example of a system of linear equation with the following properties.
   (i) Infinite number of solutions.
   (ii) No solution.

2. Write all elementary row operations.

3. Let $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find the matrix polynomial P (A) for

   $p(x) = x^2 - 2x - 3.$

4. Solve the following system by inverting the co-efficient matrix :

   $x_1 + x_2 = 2$

   $5x_1 + 6x_2 = 9.$

5. Let $W = \{(x, y) : x \geq 0, y \in \mathbb{R}\}$. Show that W is not a subspace of $\mathbb{R}^2$ over $\mathbb{R}$.

6. Write a basis for $\mathbb{R}^2$ over $\mathbb{R}$. Express (1, 2) as the linear combination of the basis.

7. Show that $\{1, x, x^2 \ldots. x^n\}$ spans P_n, where P_n is the vector space of polynomials of dimension $n + 1$.

Turn over

8. Define linearly independent set. Show that the set {(1, 0), (0, 1)} is linearly independent.

9. Find the number of parameters in the general solution of $Ax = 0$, if A is a 5×7 matrix of rank 3.

10. Find the reflection of the vector (1, 1) about the line thorough origin that makes an angle of $\frac{\pi}{6}$ with the x-axis.

11. Write all fundamental spaces of a matrix A.

12. Find the image of the unit square under the multiplication by the invertible matrix $\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$.

13. Define eigen values and eigen vectors of a matrix.

14. Find the eigen values of $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$.

15. Define an orthonormal set. Give an example.

(Ceiling 25 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 *marks.*

All questions can be attended.

Overall ceiling 35.

16. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ define by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3)$. Find the standard matrix of the operator T.

17. Let $A = \begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$ be the augmented matrix of a linear system. Find the values of a and b such that the system has a unique solution.

18. Show that the vectors $v_1 = \{1, 2, 1\}$, $v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$ form a basis of $\mathbb{R}^3$.

19. Let S = $\{v_1, v_2, \ldots v_n\}$ is a basis for a vector space V. Then show that every element $v \in V$ can be expressed in uniquely in the form of :

$$v = c_1v_1 + c_2v_2 + \ldots + c_nv_n,\ c_i \text{ are scalars for every } i.$$

20. Find the rank and nullity of the matrix :

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

21. Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$, be the reflection about y-axis and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about the x-axis. Verify that $T_1 o T2 = T_2 \text{ o } T_1$.

22. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be vectors in $\mathbb{R}^2$. Verify that

$$\langle u, v \rangle = 3u_1v_1 + 2u_2v_2,$$

satisfies all axioms of inner product.

23. Show that the basis $\left\{(0,1,0), \left(-\frac{4}{5}, 0, \frac{3}{5}\right), \left(\frac{3}{5}, 0, \frac{4}{5}\right)\right\}$ form an orthonormal basis for $\mathbb{R}^3$.

(Ceiling 35 marks)

Section C (Essay Type Question)

Answer any **two** *questions.*

Each question carries 10 *marks.*

24. (a) Solve
$$x_1 + 3x_2 + 4x_4 + 2x_5 = 0$$
$$x_3 + 2x_4 = 0$$
$$x_6 = \frac{1}{3}.$$

(b) Find the row reduced echelon form of $\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$.

Turn over

(b) Determine whether the polynomials

$$1 - x,\ 5 + 3x - 2x^2,\ 1 + 3x - x^2,$$

Are linearly independent or not.

(b) Find the Wronskian of :

$$\{1, e^x,\ e^{2x}\}.$$

26. (a) Define rank of a matrix. Find the rank of :

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}.$$

(b) Define nullity of a matrix. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined by the formula

$$T(x_1, x_2) = (x_1 + 3x_2, x_1 - x_2, x_1).$$

Find the nullity of T.

27. (a) Define Gram-Schmidt process.

(b) Use Gram-Schmidt process transform the basis.

$$\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$$

into orthonormal basis.

(2 × 10 = 20 marks)